What Classical Connectives Mean
James W. Garson

Even in the analysis of Greek and Latin (where the 'future' like the 'present' and the 'past' is realized inflexionally), there is some reason to describe the 'future tense' as partly modal.

John Lyons, *Introduction to Theoretical Linguistics*
Mild Inferentialism
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Rules determine what connectives mean.
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The rules determine truth conditions for the connectives.
Mild Inferentialism

Rules determine what connectives mean.

The rules determine truth conditions for the connectives.

Mild inferentialists allow that connective meaning may be expressed in semantical rather than purely syntactic terms.
What do natural deduction rules of propositional logic express about connective meaning?

Natural Deduction (ND) rule:

\[
\frac{H, A \vdash C}{H \vdash A \rightarrow C}
\]

Conditional Proof
What do PL rules express about connective meaning?

1. What do *sentences* express?
What do PL rules express about connective meaning?

1. What do sentences express?

Example:

$$\exists x \exists y (Fx \& \sim Fy)$$

expresses that the domain of a model contains at least two objects.
What do PL rules express about connective meaning?

1. What do sentences express?

A sentence $S$ expresses $P$ iff any model of $S$ has $P$.

$$\exists x \exists y (Fx \& \neg Fy)$$

expresses that the domain of a model contains at least two objects.
What do PL rules express about connective meaning?

A model of *sentence* is an interpretation that makes it *true*. 
What do PL rules express about connective meaning?

2. What do rules express?
What do PL rules express about connective meaning?

2. What do rules express?

A rule R expresses P iff any model of R has P.

What is the “honorific” in case of a rule?
What do PL rules express about connective meaning?

What is a model of a rule $R$?

A rule is sound (i.e. good) when it preserves validity.
What do PL rules express about connective meaning?

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A rule is sound (i.e. good) when it preserves validity.

A model of a rule preserves validity.
What do PL rules express about connective meaning?

What is preservation of validity?

A model \( V \) is a set of valuations.
What do PL rules express about connective meaning?

What is preservation of validity?

A model V is a set of valuations.

Valuations can assign any truth values they like to the wffs, but they have to make at least one f.
What do PL rules express about connective meaning?

What is preservation of validity?

A model $V$ is a set of valuations.

$H / C$ is $V$-valid iff for each $v \in V$, if $v(H) = t$ then $v(C) = t$. 
What do PL rules express about connective meaning?

What does it mean for \( V \) to be a model of a rule \( R \)?

\( V \) is a model of \( R \) iff
\( R \) preserves \( V \)-validity.
What do PL rules express about connective meaning?

What does it mean for V to be a model of a rule R?

V is a model of R iff R preserves V-validity.

If H, A ⊨ C is V-valid then H ⊨ A → C is V-valid.

Conditional Proof
What do PL rules express about connective meaning?

V is a *model of* $R$ iff $R$ preserves V-validity.

A rule $R$ *expresses* $P$ iff any model of $R$ has $P$. 
What do PL rules express about connective meaning?

Do PL rules express the meanings of the connectives they regulate?
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Do PL rules express the meanings of the connectives they regulate?

YES (Garson 1990, 2001)
What do PL rules express about connective meaning?

Do PL rules express the meanings of the connectives they regulate?

YES (Garson 1990, 2001)

The semantics expressed is a variant of a semantics for intuitionistic logic given in (Humberstone 1981).
The Semantics Expressed by PL
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$V$ set of valuations

$\leq$ Defined by:
$v \leq v'$ iff if $v(p)=t$, then $v'(p)=t$.

Think of $\leq$ as an ordering on cases.
The Semantics Expressed by PL

The “intuitionistic” truth conditions:

\[ \vDash \& \vDash \quad v(A \& B) = t \text{ iff } v(A) = t \text{ and } v(B) = t. \]
The Semantics Expressed by PL

The “intuitionistic” truth conditions:

\[ \langle A \rightarrow B \rangle = t \iff \text{for all } v' \in V, \text{ if } v \leq v' \text{ then } v'(A) = f \text{ or } v'(B) = t. \]
The Semantics Expressed by PL

The “intuitionistic” truth conditions:

\[ ||\neg|| \quad v(\neg A) = t \text{ iff } \]
\[ \text{for all } v' \in V, \text{ if } v \leq v' \text{ then } v'(A) = f. \]

Diagram:

\[ \neg A \]
\[ v \]
\[ A \]
\[ A \]
\[ A \]
\[ A \]
\[ A \]
\[ A \]
\[ A \]
The Semantics Expressed by PL

PLUS a condition for Double Negation:

\[ \text{if } v(A) = f, \text{ then for some } v' \in V, \]
\[ v'(\neg A) = t. \]

If \( A \) then

\[ \neg A \]
Open Future Semantics

There are certain events over which I have no agency, because they are already settled.
Open Future Semantics

There are certain events over which I have no agency, because they are already settled.

For example, events of the *past*:

Jim was speeding at noon Jan. 3, 1987.
*Open Future Semantics*

There are certain events over which I have no agency, because they are already settled.

Tautologies about the *future* are also settled.

\[
S \rightarrow S
\]

\[S = \text{there will a sea battle at noon tomorrow.}\]
Open Future Semantics

There are certain events over which I have no agency, because they are already settled.

Desderatum: An open future semantics should reflect these intuitions about what is settled, and what is not.
Open Future Semantics

Settled and Unsettled:

(SettledT) \( v(A) = T \) iff for all \( v' \in V \), if \( v \leq v' \), then \( v'(A) = t \).

(SettledF) \( v(A) = F \) iff for all \( v' \in V \), if \( v \leq v' \), then \( v'(A) = f \).

(Unsettled) \( v(A) = U \) iff neither \( v(A) = T \) nor \( v(A) = F \).
Open Future Semantics

Interesting Facts

(Fact 1) \( v(A) = T \) iff \( v(A) = t \).
Open Future Semantics

Interesting Facts

(Fact 1) \( v(A) = T \iff v(A) = t. \)

(Fact 2) \( v(A) = F \iff v(\neg A) = t. \)
Open Future Semantics

Interesting Facts

(Fact 1) $v(A) = T \iff v(A) = t$.

(Fact 2) $v(A) = F \iff v(\neg A) = t$.

(Fact 3) $v(A) = U \iff$

$v(A) = f$ and $v(\neg A) = f$. 
Open Future Semantics

A is \textit{settled at v} iff $v(A) \neq U$.

(Fact 4) $v(A) \neq F$ iff $v(A) = T$, if A is settled at v.
Open Future Semantics

A is settled at \( v \) iff \( v(A) \neq U \).

(Fact 4) \( v(A) \neq F \) iff
\[ v(A) = T, \text{ if } A \text{ is settled at } v. \]

\[ v(A) = qT \text{ (quasi true)} \text{ iff} \]
\[ v(A) = T, \text{ if } A \text{ is settled at } v. \]
Open Future Semantics

\[ v(A) = qT \quad (\text{quasi true}) \text{ iff } \]
\[ v(A) = T, \text{ if } A \text{ is settled at } v. \]

A is \textit{weakly inevitable} at v iff
for all v’, if v \leq v’, then v’ (A) = qT.
Open Future Semantics

Puzzle:

If $v(A) = f$ then for some $v' \in V$, $v'(\neg A) = t$.

What does this mean?
Open Future Semantics

Puzzle:

$\lll \text{if } v(A) = f \text{ then for some } v' \in V, v'(\neg A) = t. \lll$

$\lll$ is equivalent to:

If $A$ is inevitable at $v$, then $v(A) = T.$
Open Future Semantics

If A is inevitable at v, then v(A)=T.

Example:  S→S
S = a sea battle will occur tomorrow.

This is settled true because it is inevitable.
Open Future Semantics

If $A$ is inevitable at $v$, then $v(A)=T$.

Example: $S\rightarrow S$
$S = a$ sea battle will occur tomorrow.

This is settled true because it is inevitable ... even though $S$ is not settled.
Open Future Semantics

If A is inevitable at v, then v(A)=T.

Example:  S→S  
S = a sea battle will occur tomorrow.

This is settled true because it is inevitable ... even though S is not settled.

So ||LL|| matches intuitions about where we lack agency.
Disjunction Truth Condition:

Garson (1990) reports a condition that the $v$ rules express.

$\|Cv\| v(A\lor B)=t$ iff for every wff $C$, if $v(C)=f$, then there is a $v'$ in $V$ such that $v\leq v'$, $v'(C)=f$, and either $v'(A)=t$ or $v'(B)=t$. 
Disjunction Truth Condition:

Garson (1990) reports a condition that the \( v \) rules express.

\[ \|C\| v(A \lor B) = t \text{ iff for every wff C, if } v(C) = f, \text{ then there is a } v' \text{ in } V \text{ such that } v \leq v', v'(C) = f, \text{ and either } v'(A) = t \text{ or } v'(B) = t. \]

This is arguably circular, and so not a truth condition.
Disjunction Truth Condition:

In the presence of \( \|LL\| \), this condition reduces to one that is well behaved.
Disjunction Truth Condition:

\[ v(A \lor B) = t \text{ iff for all } v', \text{ if } v \leq v', \text{ then } v'(A) = qT \text{ or } v'(B) = qT. \]

\( A \lor B \) is \( t \) iff in every future, one of its disjuncts is \( qT \).
Disjunction Truth Condition:

\[ v(A \lor B) = t \iff \text{for all } v', \text{ if } v \leq v', \text{ then } v'(A) = qT \text{ or } v'(B) = qT. \]

So \( A \lor \neg A \) is \( t \iff \) in each future \( A = qT \) or \( \neg A = qT. \).
Disjunction Truth Condition:

The upshot:

Av~A is settled true at v even if neither disjunct is settled at v.
Disjunction Truth Condition:

The upshot:

$Av\neg A$ is settled true at $v$ even if neither disjunct is settled at $v$.

This matches open future intuitions.
Supervaluations

Supervaluations also provide an alternative three valued semantics for PL where $A \lor \neg A$ is valid even though neither $A$ nor $\neg A$ is settled.
Supervaluations

Supervaluations also provide an alternative three valued semantics for PL where A ∨ ~A is valid even though neither A nor ~A is settled.

So what is the connection between supervaluations and the semantics expressed by PL?
Supervaluations

Points of similarity:
Supervaluations

Points of similarity:

1) Partial truth tables are the same.
Supervaluations

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1) Partial truth tables are the same.

<table>
<thead>
<tr>
<th>A</th>
<th>T</th>
<th>F</th>
<th>U</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>A→B</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>T</td>
<td>T</td>
<td>F</td>
<td>N</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>U</td>
<td>T</td>
<td>N</td>
<td>T/N</td>
</tr>
</tbody>
</table>
Supervaluations

Points of similarity:

1) Partial truth tables are the same.

2) There is an isomorphism between the set of all supervaluations and the canonical model for PL.
Supervaluations

Points of difference:
Supervaluations

Points of difference:

1) Supervaluations do not provide truth conditions for the connectives.
Supervaluations

Points of difference:

1) Supervaluations do not provide truth conditions for the connectives.

2) There is no open futures structure $\leq$. 
Supervaluations

Points of difference:

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3) Supervaluation semantics is not sound for MT, CP, IP vOUT!
Supervaluations

Points of difference:

1) Supervaluations do not provide truth conditions for the connectives.

2) There is no open futures structure.

3) Supervaluation semantics is not sound for MT, CP, IP vOUT!

PL does not express supervaluations!
What do we mean by ~, →, and v?

Suppose alien anthropologists study our logic to determine what we mean by the connectives.
What do we mean by \( \neg, \to, \text{ and } \lor \) ?

Suppose alien anthropologists study our logic to determine what we mean by the connectives.

If they look at *uses* embodied by our rules’ preservation of validity, they MUST believe we mean the truth conditions for an open future.
What do we mean by \( \sim, \to, \text{ and } \lor \)?

Suppose alien anthropologists study our logic to determine what we mean by the connectives.

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*Mustn’t we draw the same conclusion?*
Application to Vagueness:
(with Joshua D. K. Brown)

Williamson’s (1994) complaints against supervaluations.
Application to Vagueness:

Williamson’s complaints against supervaluations.

1. Supervaluations are not sound for PL.
**Application to Vagueness:**

Williamson’s complaints against supervaluations.

1. Supervaluations are not sound for PL.

**Answer:** Natural semantics is (by definition) sound for PL.
Application to Vagueness:

Williamson’s complaints against supervaluations.

2. Supertruth is not disquotationnal.

Answer: In natural semantics t=T, so truth just is “supertruth”.
Application to Vagueness:

Williamson’s complains against supervaluations.

3. Supervaluations validate \( \exists x (Bx \& \neg Bx + 1) \).

Answer: In natural semantics precisification is partial (thanks to \( \leq \)) so \( \exists x (Bx \& \neg Bx + 1) \) can be falsified.
Application to Vagueness:

Williamson’s complains against supervaluations.

4. Supervaluations cannot account for higher-order vagueness.

Answer: We have independent evidence that natural semantics should be deployed at every level.
Application to Vagueness:

Natural semantics is a natural account for rejecting epistemicist accounts of vagueness.

Indeterminacy runs deep, for it is built right into the meanings for the connectives expressed by the classical rules for PL.